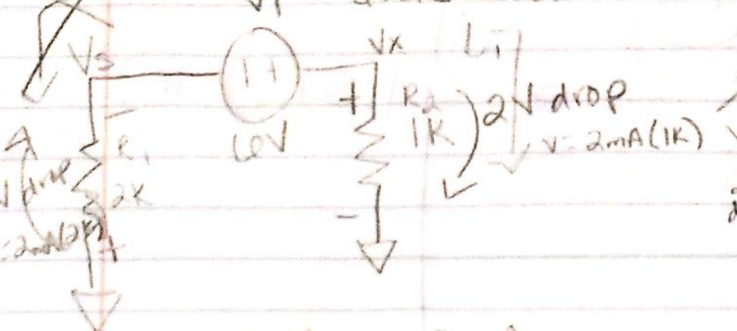
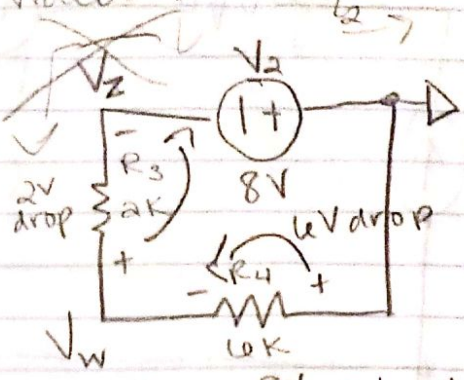
Superposition Notes

Quiz Review in video:





$i_1 = \frac{6V}{3K} = 2mA$

$i_2 = \frac{8V}{8K} = 1mA$

$V_s = 0 - 4 = -4$ $V_x = 2V = 0 - -2V = 2V$

$V_w = 0 - 6 = -6V$

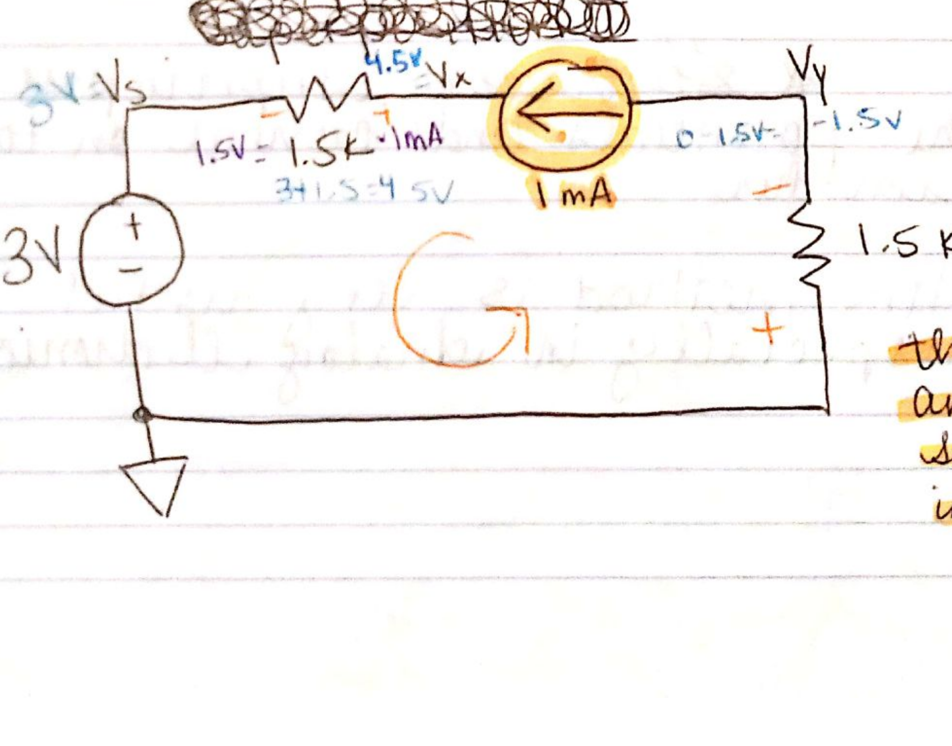
$V_z = 0 - 8 = -8V$

* Active components will not pass current through itself

Passive components will only pass current through itself

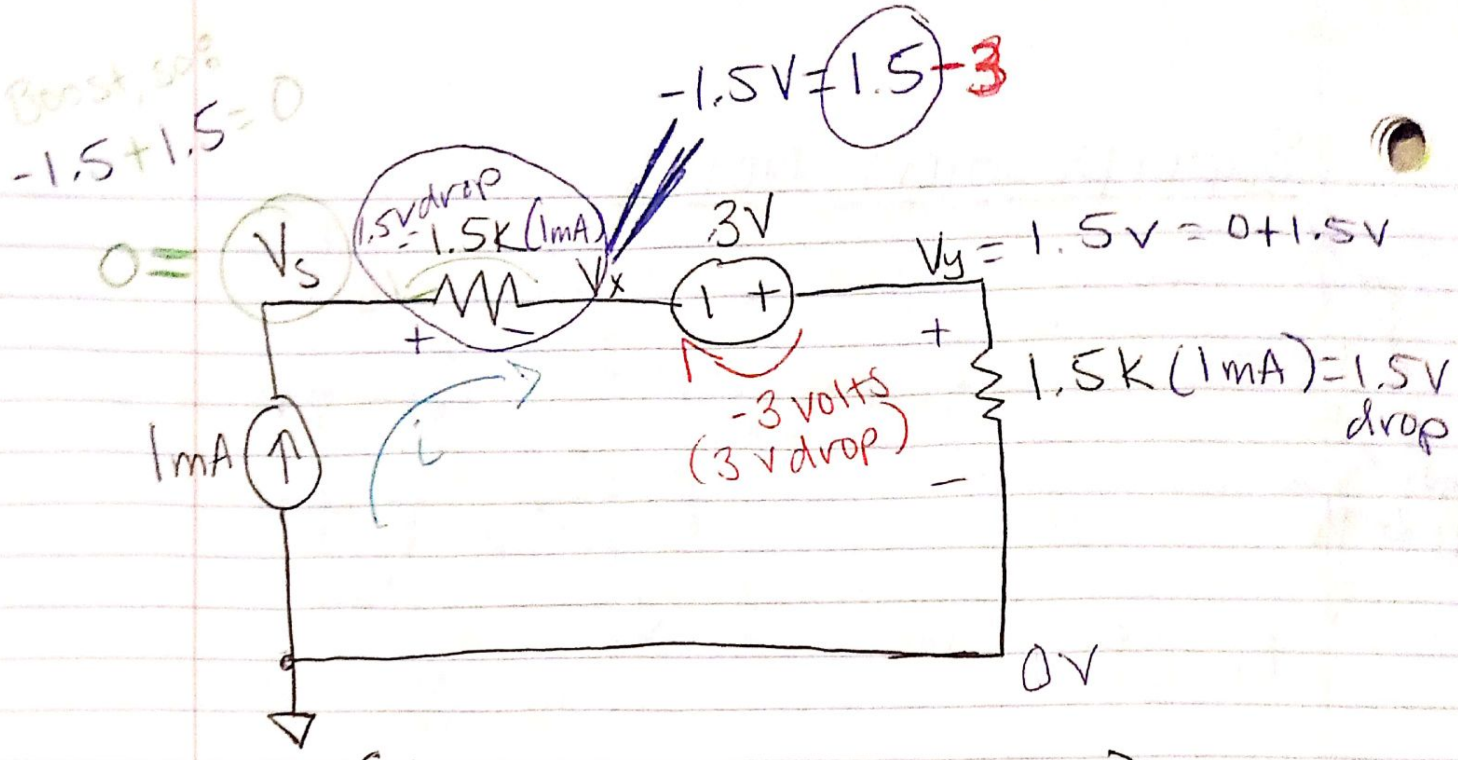
voltage of a node relies on the voltage drop over the passive component

More Review



Current sources will truncate current value in ideal situations

they also have an external power supply not shown in the diagrams!



(Superposition Theorem)

Superposition I

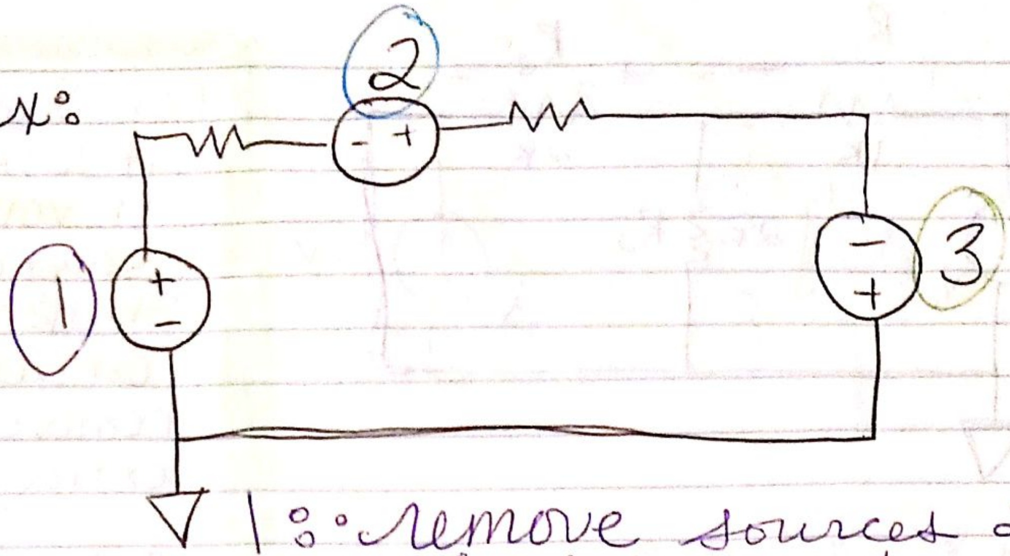
Analysis method:

★ Eliminate all but one source of power within a network at the time

★ Calculate the potential drops and current flows through the components

★ Algebraically superimpose the potentials and current on top of eachother

This method is very useful (especially in Analog electronics) ex:



1: · remove sources 2 and 3
· Calculate current flows & potential drops through all components

· Proceed to 2

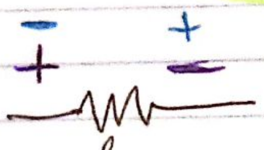
2: remove all sources except source 2

· calculate trough all components that aren't the power sources

· Proceed to 3

3: · Repeat steps
· Superimpose the values

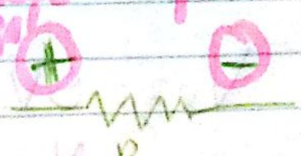
ex:



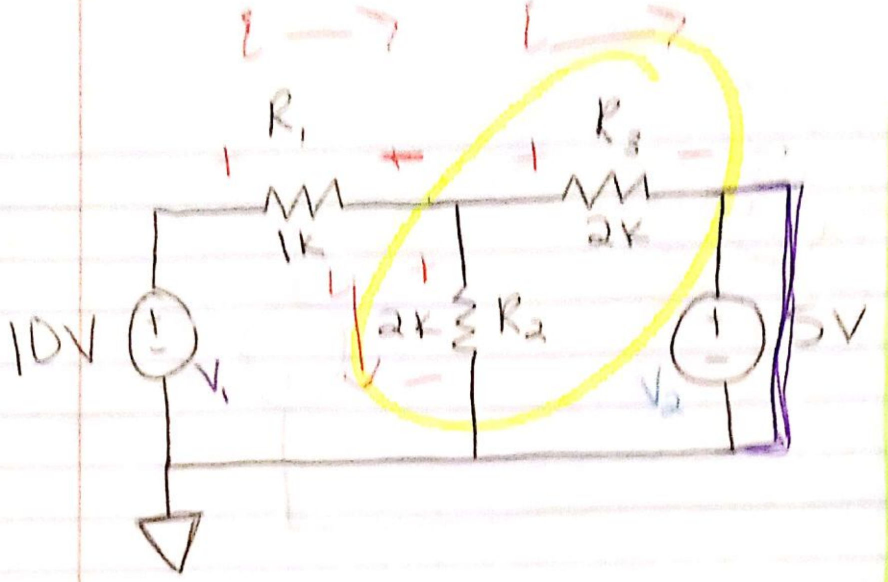
1: 5V
2: 4V

takes direction of highest potential diff

Superimposed:



direction now changed → 5V + (-4V) = 1V



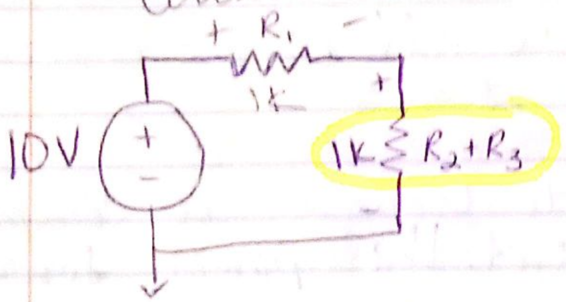
For Superpos, if looking at voltage source, SHORT the circuit to remove the extra sources;

if looking at current sources, disconnect the circuit

V_1 Only: define current direction and polarity

in ∥ $\quad \frac{2k \cdot 2k}{2k+2k} = \frac{4k}{4k} = 1k$

Circuit now:



$V_{R1} = \frac{10V}{2} = 5V$ $\quad$ + 1k − $R1$

$i_{R1} = \frac{V_{R1}}{R1} = \frac{5V}{1k} = 5mA$

Voltage divider: $10V \cdot \frac{1k}{2k} = 10v \cdot \frac{1}{2} = \frac{10V}{2} = 5V$

+ R2 2K −

$V_2 = 5V$ bc R1 has 5V drop

$\boxed{i_{R2} = \frac{V_{R2}}{R2} = \frac{5V}{2K} = 2.5\,mA}$ R2 connects to GND (0V)

$10V - 5V - V_{R2} = 0$

$-5V + V_{R2} = 15V$

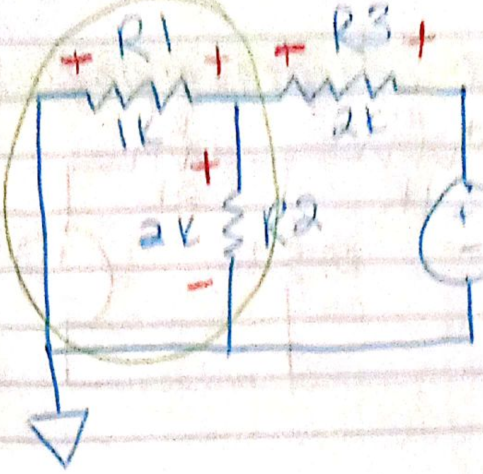
$-5V$

+ R3 − 2K

R3: $V_{R3} = 5V$ b/c in ∥

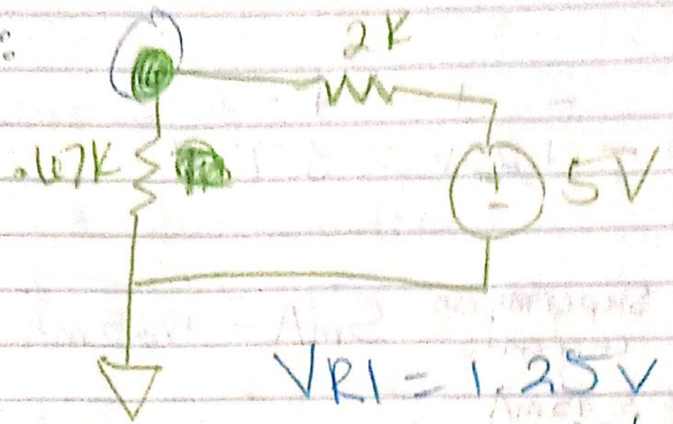
$i_{R3} = \frac{V_{R3}}{R3} = \frac{5V}{2K} = 2.5mA$

V2 only:



$\frac{1K \cdot 2K}{1K+2K} = \frac{2}{3} = 0.67$

V_{R1}:



$5V\left(\frac{0.67}{2K+0.67K}\right)$

$= 5V\left(\frac{.67K}{2.67K}\right)$

$= 1.25V$

$V_{R1} = 1.25V$

$= 0+1.25V$ 0

R1 +

1.25

$i_{R1} = \frac{V_{R1}}{R1} = \frac{1.25V}{1K} = 1.25mA$

1.25V

$V_{R2} = 1.25V$

+ R2 1.25V drop

−

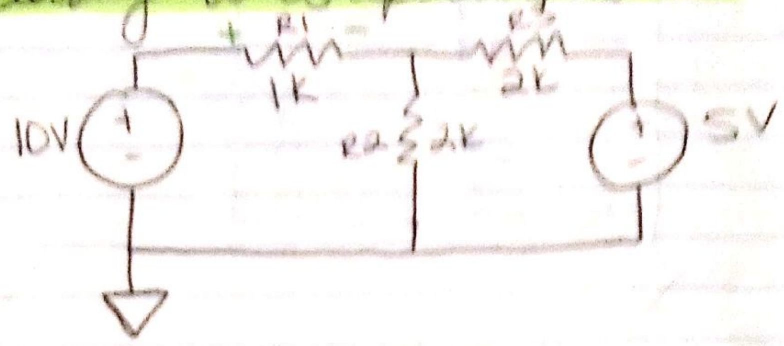
0V

$i_{R2} = \frac{V_{R2}}{R2} = \frac{1.25V}{2K} = .625\,mA$

$V_{R3} = 5V - 1.25V = 3.75V$

$i_{R3} = \frac{V_{R3}}{R3} = \frac{3.75V}{2K} = 1.875mA$

Ready to Superimpose:



R1: $5V - 1.25V = 3.75V$

i_{R1}: $\frac{V_{R1}}{R_1}$ or superimpose currents $5mA - 1.25mA = 3.75mA$

$\frac{3.75V}{1K} = 3.75mA$

R2: $5V + 1.25V = 6.25V$

i_{R2}: $\frac{6.25V}{2K} = 3.125mA$

add them b/c same current direction for both!

R3: $5V - 3.75 = 1.25V$

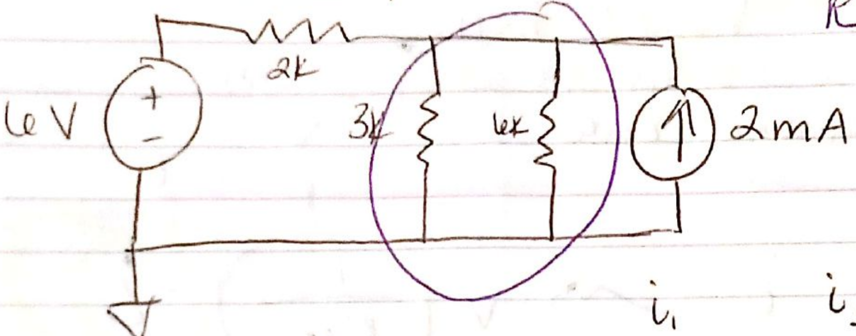
i_3: $2.5mA - 1.875mA = 0.625mA$

Superposition II : review notes on back (very helpful!)

rest of video reworks the example in Superposition I lecture AND verifies the solution in LTSpice

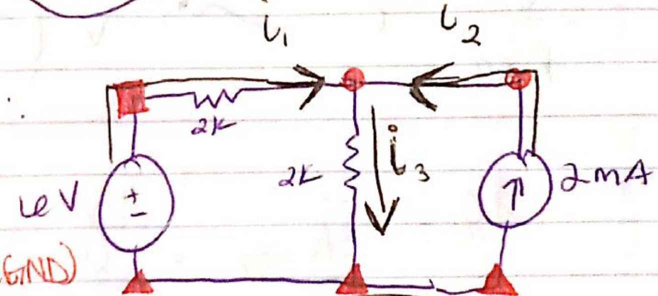
Superposition III :

Review problem :



$$R_{eq} = \frac{3k \cdot 6k}{3+6k} = \frac{18k}{9k} = 2k$$

1: redraw:



2: label nodes

■ = 6V ▲ = 0V (GND)

● = Vx

equations:

Kirchoff's Current Law

1) $i_1 + i_2 = i_3$

2) $i_1 = \frac{V}{R} = \frac{6V - V_x}{2k} =$

3) $i_2 = 2mA$ (from current source)

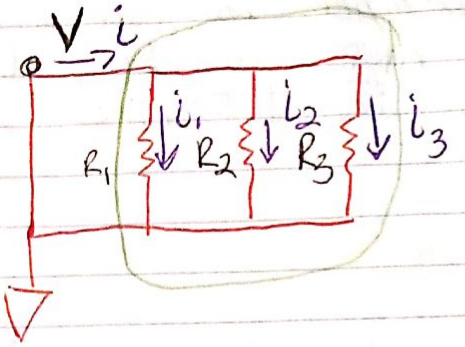
4) $i_3 = i_1 + i_2 = \left(\frac{6V - V_x}{2k}\right) + 2mA$ OR $\frac{V_x - 0}{2k}$

Ohms Law: $V\left(\frac{1}{R_{eq}}\right) = I$

Why $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$

(for $\parallel$ only)

ex



K. current Law says $i = i_1 + i_2 + i_3$

Ohms Law says

$V = i(R)$

or

$\frac{V}{R} = i$

Req b/c V and i both are divided w/ all 3*

*applying Ohm's Law

$$\frac{V}{R_{eq}} = i \rightsquigarrow V\left(\frac{1}{R_{eq}}\right) = i$$

So... $\frac{1}{R_{eq}}$! yay!! Thank you Dr. Li!

* Req is like treating the $\parallel$ resistors as a black box

$i_1 = \frac{V}{R_1}$ $\quad i_2 = \frac{V}{R_2}$ $\quad i_3 = \frac{V}{R_3}$

$$i = V\left(\frac{1}{R_1}\right) + V\left(\frac{1}{R_2}\right) + V\left(\frac{1}{R_3}\right)$$

Superposition III (continued)

$i_1 + i_2 = i_3 - i_1$

$(2k)\ 2mA = \frac{V_x}{2k} - \frac{6V - V_x}{2k}\ (2k)$

$4mA = V_x - 6V - V_x$

$4mA = -6V - 0$

$+6V \quad +6V$

$10V$

Watch your Algebra

3: find currents

$i_1 + i_2 = i_3$

$i_1 = \frac{6V - V_x}{2k}$ $\quad$ $i_2 = 2mA$ $\quad$ $i_3 = \frac{V_x - 0}{2k}$

$= \frac{6V - 5V}{2k} = \frac{1V}{2k} = .5mA$ $\quad$ $\frac{5V - 0V}{2k} = \frac{5}{2}$

$= 2.5$

$(\cancel{2k})\ \frac{6V - V_x}{\cancel{2k}} + 2mA = \frac{V_x - 0}{2k}\ (\cancel{2k})$

$i_1 = .5mA \quad i_2 = 2mA \quad i_3 = 2.5mA$

$6V - \cancel{V_x} + 4V = V_x - 0$

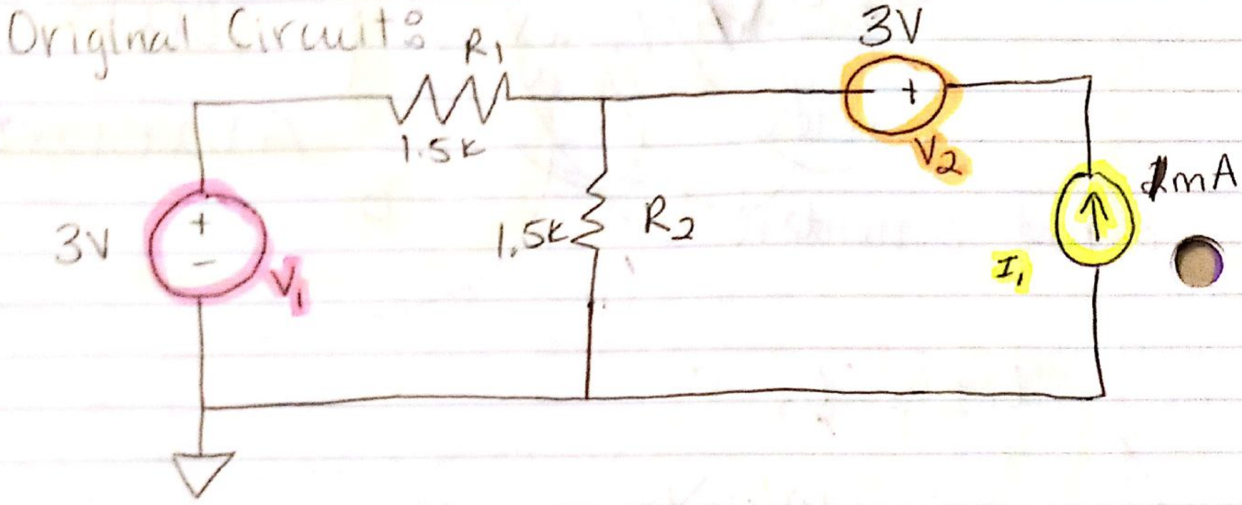
$+V_x \qquad +V_x$

$\frac{10V}{2} = \frac{2V_x}{2} \rightarrow V_x = 5V$

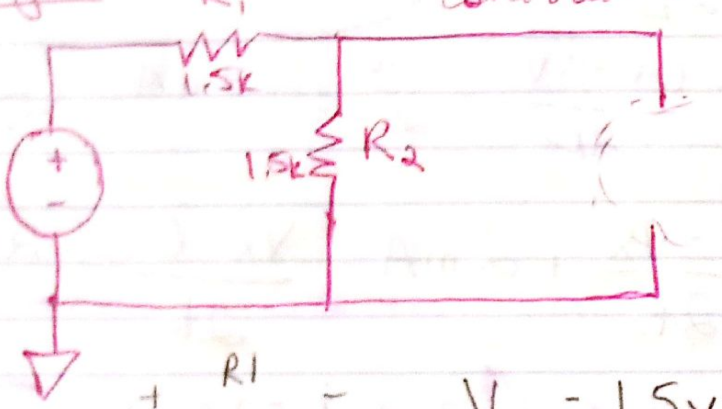
Convert circuit to a simpler one,
do calculations,
then convert it back (and forth)
when needed

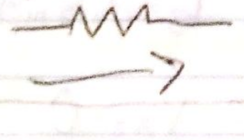
Next Example:

Original Circuit:



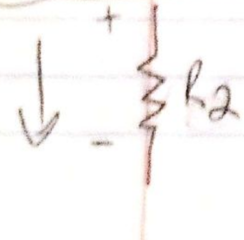
V1 Only:





$V_{R_1} = 1.5v$ voltage divider!

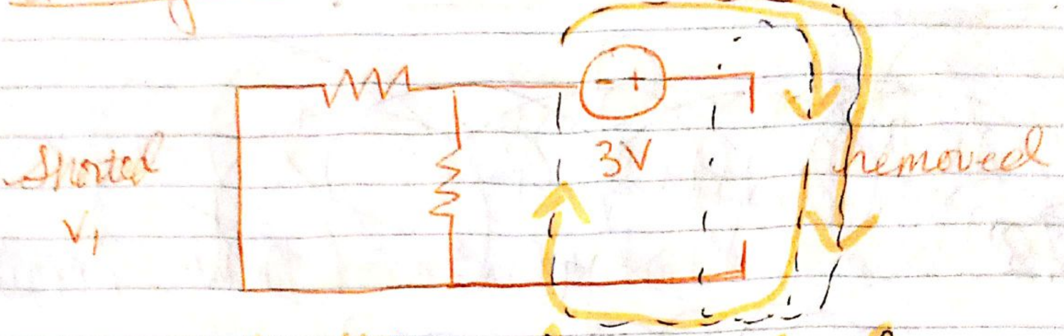
$i_{R_1} = \frac{V_{R1}}{1.5k} = \frac{1.5V}{1.5k} = 1mA$



$V_{R2} = 1.5V$

$i_{R2} = \frac{V_{R2}}{1.5k} = \frac{1.5V}{1.5k} = 1mA$

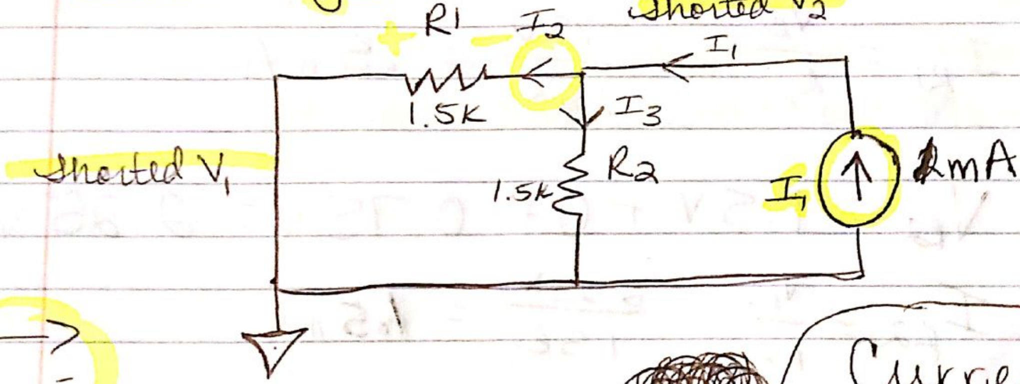
V_2 only:

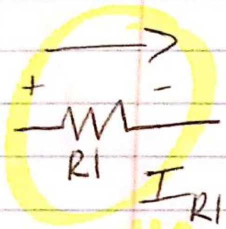


Nothing happens here, no arc will form to complete the circuit, so all values are 0 for this version of the circuit

:)

I_1 only:

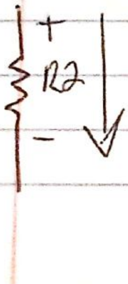




$V_{R1} = I_{R1} \cdot R_1 = -.5mA(1.5k) = -.75mA$

$I_{R1} = -.5 mA$

opposite direction!



$V_{R2} = I_{R2} \cdot R_2 = .5mA(1.5k) = .75mA$

$I_{R2} = 0.5mA$

Current Divider

$I_2 = I_1 \left(\frac{R_2}{R_1+R_2}\right) = 1mA\left(\frac{1.5k}{1.5k+1.5k}\right) = 1\left(\frac{1.5k}{3k}\right) = .5mA$

$I_3 = I_1 \left(\frac{R_1}{R_1+R_2}\right) = 1mA\left(\frac{1.5k}{1.5k+1.5k}\right) = 1\left(\frac{1.5k}{3k}\right) = .5mA$

Now Superimpose:

$$V_{R_1} = 1.5V + 0 + (-.75V) = 0.75V$$

$$I_{R1} = \frac{V_{R1}}{R_1} = \frac{.75V}{1.5k} = .5\,mA$$

$$V_{R2} = 1.5V + 0 + 0.75V = 2.25V$$

$$I_{R2} = \frac{V_{R2}}{R2} = \frac{2.25V}{1.5k} = 1.5\,mA$$

Reason behind short vs. disconnect:

Ideally, internal resistance of a Voltage source is 0, so shorting the wires does not hinder Req

The internal resistance of a Current source is ~~finite~~ Infinite, so shorting the wires to remove would be inconsistent to the original Circuit

So always short the circuit to remove the voltage source

and open the circuit to remove a current source.

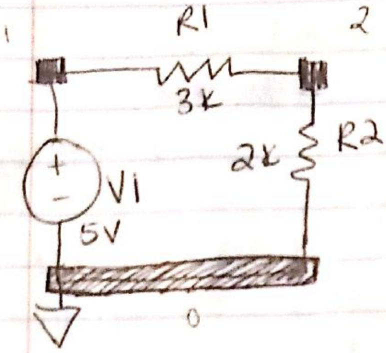
SPICE Code Refresher

Circuit design:

task

component name	High node	Low node	A/DC type	quantity
V1	1	0	dc	5V
R1	1	2	NO SPACE	3K
R2	2	0		2K

this code →



← makes this circuit

SPICE Commands

t for time domain

command	start	stop	interval (sec)
.tran	0	1	.1

transient analysis in time domain

dc for voltage domain

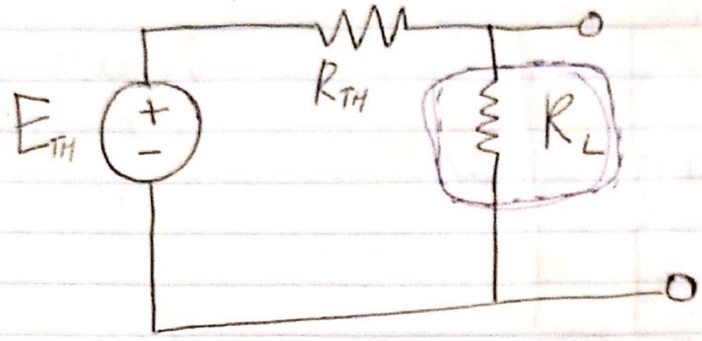
	source	start	stop	interval (v)
.dc	V1	0	10	.01

dc sweep (voltage domain)
(linear sweep by default)
sweep will ignore user set value to a power supply and simulate whatever is put into sweep command parameters

Watch before lab on Tuesday morning!

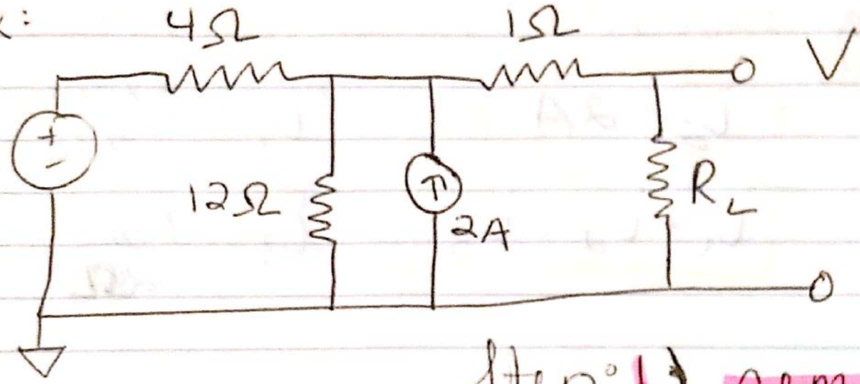
Thevenin's Equivalent

Any linear circuit can be simplified to:

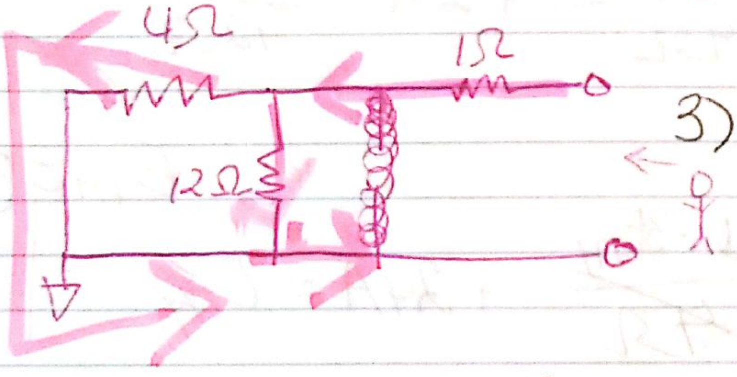


load & can be treated as a black box

ex:



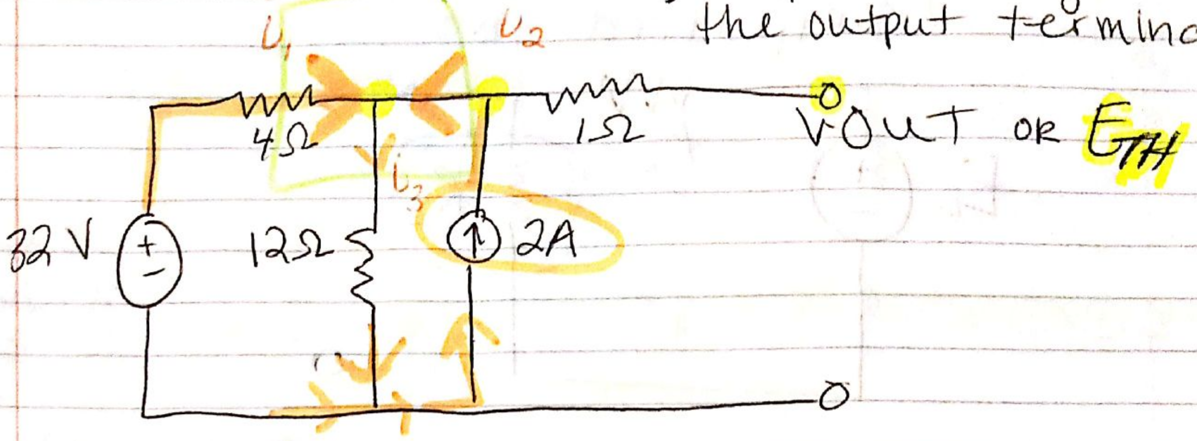
Step: 1) remove the load
2) short voltage source, disconnect current source
3) solve for R_{TH}



$$R_{TH} = 1\Omega + (4\Omega \parallel 12\Omega)$$

$$= 1\Omega + \frac{4(12)\Omega}{(4+12)\Omega} = \frac{48\Omega}{16\Omega} + 1\Omega = 4\Omega$$

E_{TH}: Step: 1) Remove R_L
2) E_{TH} is voltage at the output terminal



using current node method:

$i_2 = 2A$ $\qquad$ $i_1 = \frac{32 - E_{TH}}{4\Omega}$

$i_1 + i_2 = i_3$ $\qquad$ $i_3 = \frac{E_{TH}}{12\Omega}$

So:

$$\frac{32 - E_{TH}}{4\Omega} + 2A = \frac{E_{TH}}{12\Omega}$$

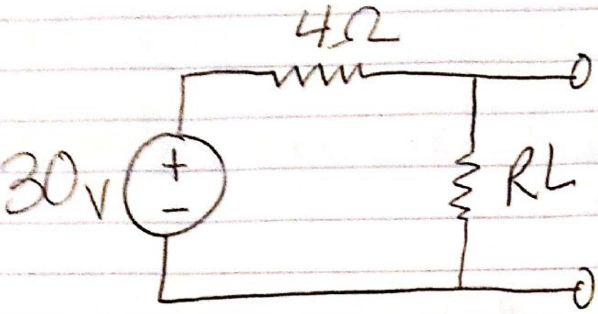
$E_{TH} = 30V$

$\rightarrow 96 - 3E_{TH} + 24A = E_{TH}$

$\rightarrow 120 - 3E_{TH} = E_{TH}$

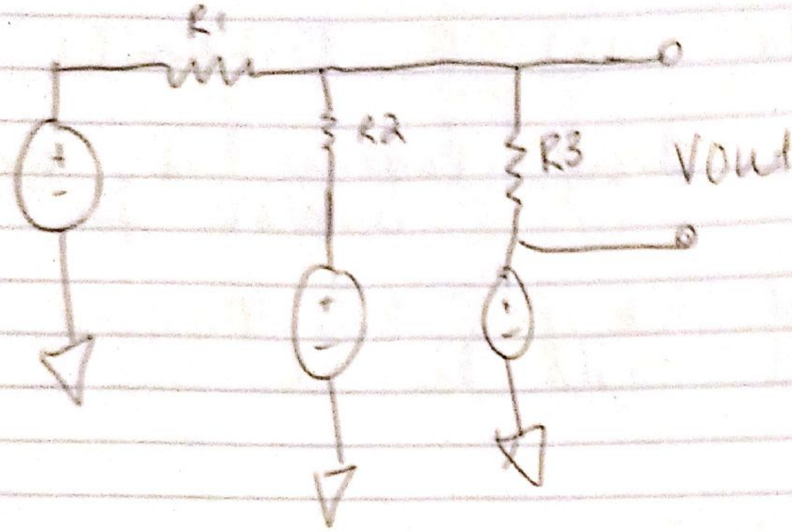
$\rightarrow \frac{120}{4} = \frac{4E_{TH}}{4}$

So, the Thevenin's equivalent circuit for this circuit is



Quick Review

Superposition -



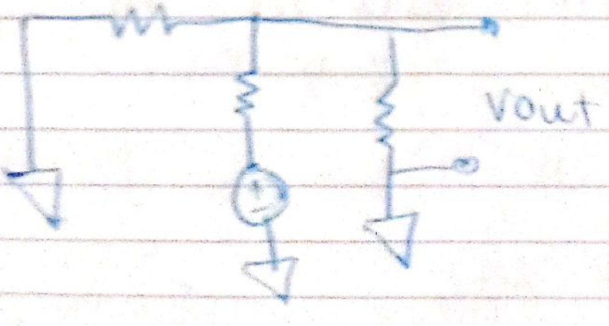
V1 only:

$R_2 \parallel R_3$

now it's a voltage divider.

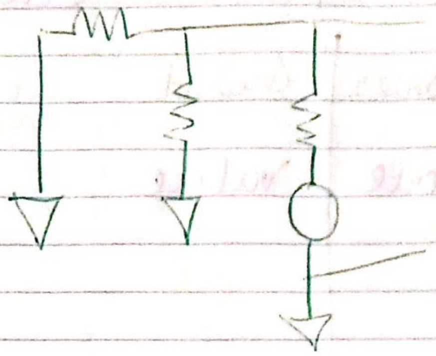
$$V_{out} = V_1\left(\frac{R_2 \parallel R_3}{R_1 + R_2 \parallel R_3}\right)$$

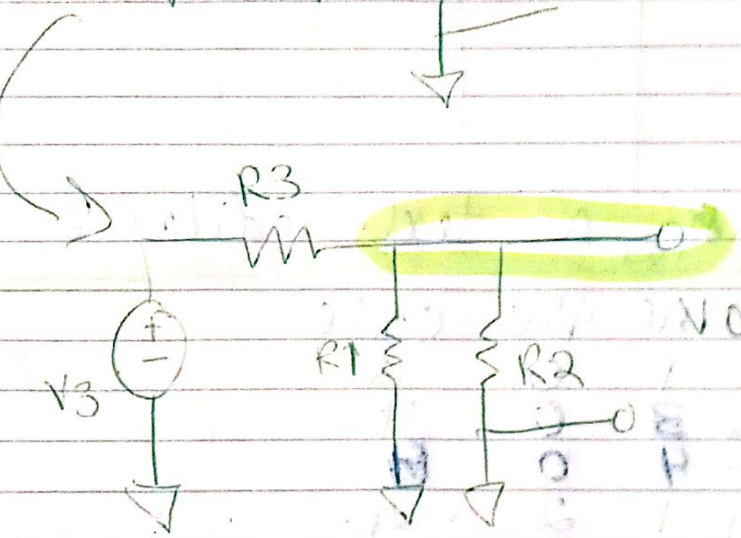
V2 only:



$$V_{out} = V_2\left(\frac{R_1 \parallel R_3}{R_2 + R_1 \parallel R_3}\right)$$

V3 only:





Same terminal, still Vout

$$R2 \parallel R3 = \frac{R2 R3}{R2 + R3}$$

$$V_{out} = V3\left(\frac{R1 \parallel R2}{R3 + R1 \parallel R2}\right)$$

now super impose

$$V_{out} = V_{out1} + V_{out2} + V_{out3}$$

$$V_{out} = V_1\left(\frac{R2 \parallel R3}{R1 + (R2 \parallel R3)}\right) + V2\left(\frac{R1 \parallel R3}{R2 + (R1 \parallel R3)}\right) + V3\left(\frac{R1 \parallel R2}{R3 + (R1 \parallel R2)}\right)$$

ex with values: $V1 = 2V$ $V2 = 4V$ $V3 = 6V$
$R1 = 2k$ $R2 = 4k$ $R3 = 6k$

Spice Code

Component	Node	Node		
V1	1	0	Quant	code

Component	higher	lower	Quant
name	node	node	value

just use a text editor:
code.txt

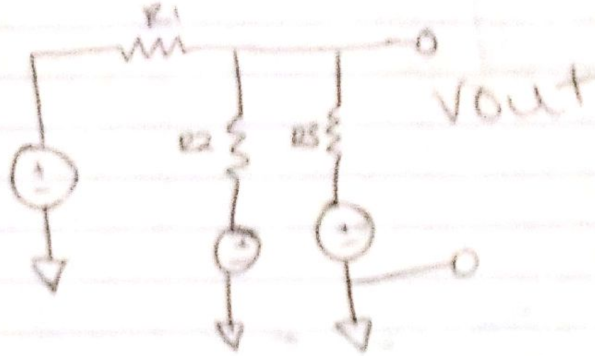
Name the code

V1	1	0	2
V2	3	0	4
V3	4	0	4
R1	1	2	2K
R2	2	3	4K
R3	2	4	6K

open txt file in LT Spice

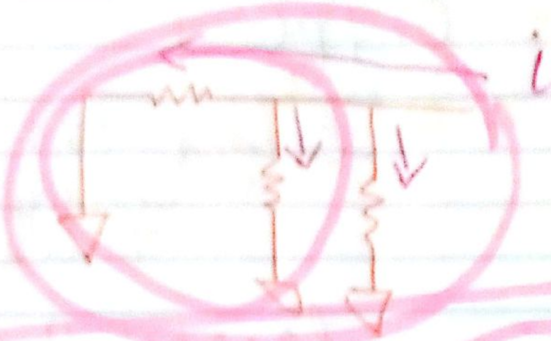
Thevenin's Equivalent

converting the same circuit



1. look backwards and remove power sources

2.



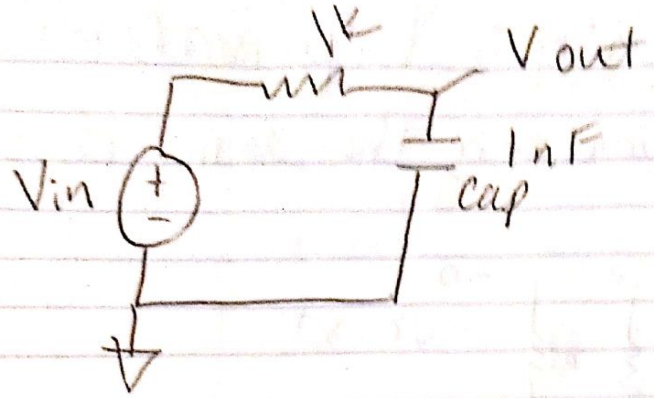
$R_{TH} = R3 \parallel (R_1 \parallel R_2)$

0-5 pulse voltage is fine

• pulse low V high V time delay rising time falling time d time

period

if:



$$\boxed{t_d = 0.7RC}$$

larger the R, the larger the time delay

larger C has more to fill

$\tau = RC$ (time constant)

$R \cdot C$ = time (micro seconds)